**Astronomy 84: Black Holes**

***Free Fall to a compact object***

**Discussion 7.1 Fall 2017**

On pages 171 and 172 of Thorne’s book, he talks about how stars may collapse to create neutron stars and how gravity drives this process. See if you can verify Thorne’s claim at the top of page 173 about the collapse of a white dwarf sized object (Earth) to a neutron star sized object.

Your group should work through the following calculations and present your findings to the class leading a discussion of gravitational free fall. These ideas are relevant to many regimes in astronomy, not just the collapse of stars but also the collapse of large clouds of gas and dust to make new stars.

When stars collapse, they do so rapidly. It is interesting to see how long it would take a star to collapse once all its fuel is exhausted. The collapse time (often called the *free-fall* time) gives us insight into the formation of white dwarfs, neutron stars, and the way supernovae are triggered.

We can estimate the collapse time for an ordinary star to reach a new configuration (e.g. the Sun collapsing from its current radius to a white dwarf of much smaller radius) by looking at the free fall times for particles falling onto a spherical mass. A particle of mass *m* that falls to distance *r* from an object of mass *M* along a parabolic trajectory, meaning it starts with zero velocity at its starting distance *r*0 from *M*, acquires a kinetic energy equal to the negative potential energy:

*½ mv*2 *=* G*Mm*/*r* – G*Mm*/*r*0 (1)

Since the velocity is just the time derivative of the distance from the star, *r*, it is possible to solve the equation of motion exactly (note that d*r*/d*t* is negative):

*v =* d*r*/d*t* = –(2G*M*)1/2 (1/*r* – 1/*r*0)1/2 (2)

Notice that the velocity does not depend on the mass of the particle. That means we think about the surface of a star as a set of test particles that fall toward the star’s center attracted by all the interior mass *M*.

1. Integrate equation (2) to find *r*(*t*) for a particle in free fall. Put the term containing *r* and *r*0 on one side and the terms in *t* on the other, then integrate both sides (the easiest way to do this is to look up the answer in a book of integrals, since this is a simple form, or see my note on at the end of this assignment).
2. Use this solution to find the time it takes a particle to fall from an initial radius, *r*0, to a final radius, *r*f, where *r*0 is much larger than *r*f. The fact that *r*0 is very large should allow you to simplify your equation by neglecting small terms.
3. Where is most of the time spent during this collapse, at large distances or small distances?

Now use these results to calculate the collapse time for three situations:

1. Collapse of a star with one solar mass and one solar radius (*R* = R🞊 = 7x108 m) to the radius of a white dwarf (*R* = *R*E = 6000 km).
2. Collapse of a solar mass star from a white dwarf size (Earth size) to a neutron star size thus checking Thorne’s statement.
3. Collapse of a solar mass star from one solar radius to black hole (*R* = *R*sh = 3 km).

A solar mass star will generally go through a giant phase where *R* ~ 1 AU (1.5x1011 m) or more before it collapses, but most of the mass is actually confined to smaller radii. Nevertheless, you should think about how the free fall times depend on the initial radius to decide if this could be an important effect.

Does it matter for your calculation that the collapsing mass is actually extended? In other words, the calculation you did above to find the free fall times assumed all the mass was concentrated in a radius equal to or smaller than the final free fall radius. But a collapsing star is actually extended, and the different radii all collapse together. Does this make a difference to your estimates? If all the material at each radius within the star started collapsing at the same time, would it all reach the center (or a new configuration) at the same time, too, or would some of the material get there faster than other material?

Does your estimate for part (e) make sense in light of your knowledge of general relativity? What aspects of general relativity have we neglected in pushing our calculation close to the Schwarzschild radius (or more properly, circumference)?

Here is a way to find the integral:

I = ∫–(1/*r* – 1/*r*0)–½ d*r*

Let 1/*r* = *x* and d*r* = –1/*x*2 d*x* then:

∫–(1/*r* – 1/*r*0)–½ d*r* = ∫x-2 (x – 1/*r*0)–½ d*x* = *r*0 *x*–1 (*x* – 1/*r*0)½ + *r*03/2 arctan[ (*r*0 *x* – 1)½ ]

= *r*0 *r* (1/*r* – 1/*r*0)–½ + *r*03/2 arctan[ (*r*0/*r* – 1)½ ]